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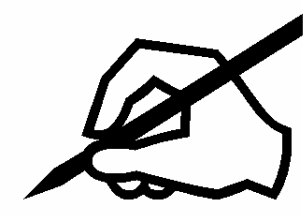

# LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT Eight
Functions and Relations

Lesson
16

# Combining Functions

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1 and 2.

(Some helpful exercises and page numbers are given throughout the lesson, e.g. (Ex 14c page 812)

## INTRODUCTION

In this lesson we will combine functions to make new ones. The commutative and associative laws will also be investigated to determine if they apply to composition of functions.

**OBJECTIVES**

At the end of this lesson you will be able to:

Interpret and evaluate composite functions

### 8.4 Composition of Functions

Functions can be combined by putting one function into the other.

Example: Given $f: x \to 3x - 2 \; and \; g: x \to 2x + 5$ .

Determine $fg(x)$.

Solution: To obtain $fg(x)$ we have to put $g$ into $f$ as follows:

$$fg(x) = 3(2x + 5) - 2$$

$$= 6x + 15 - 2$$

$$= 6x + 13.$$

**Note composite functions are not commutative**

$$gf(x) = 2(3x - 2) + 5$$

$$= 6x - 4 + 5$$

$$= 6x + 1 \; .$$

$$fg(x) \neq gf(x)$$

**Composite Functions are associative**

Example: Given

$f(x) = 3x, \quad g(x) = x + 1, \quad and \; h(x) = 2x - 1, \quad determine:$

a) $[f(gh)](x)$

b) $[(fg)h](x)$

Solution: a) $gh = 2x - 1 + 1$

$= 2x$

$[f(gh)](x) = 2(3x)$

$= 6x$

b) $[(fg)h](x)$

$fg = 3(x + 1)$

$fg = 3x + 3$ .

$(fg)h = 3(2x - 1) + 3$

$= 6x - 3 + 3$

$[f(gh)](x) = 6x$

Therefore $[f(gh)](x) = [(fg)h](x)$.

**Evaluating composite functions**

Example:

The functions $f \text{ and } g$ are defined by:

$f(x) = \frac{2x + 3}{x - 1}$ and $g(x) = 3x + 1$ .

Determine the value of $fg\left(\frac{1}{3}\right)$.

Solution: Determine $g\left(\frac{1}{3}\right)$ then substitute this value into $f$ .

$$g\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right) + 1$$

$$= 1 + 1$$

$$= 2.$$

$$f(2) = \frac{2(2)+3}{2-1}$$

$$= \frac{4+3}{1}$$

$$= 7.$$

**ACTIVITY 1**

Given that $g(x) = \frac{3x-1}{x+2}$ *and* $h(x) = \frac{7x+1}{5x-1}$

(a) Evaluate $g(3)$

(b) Show that $hg(3) = \frac{61}{35}$

(Ex 14c page 812)


CXC question

Given that $g(x) = x + 3$ *and* $h(x) = x^2$,

calculate $hg(0)$.

**Conclusion**

We looked at combining functions and showed that while this is associative it is not commutative. In the lesson that follows we will work with inverse of a function. This includes the inverses of composite functions.